2’s Complement

Introduction

Do negative numbers exist? Did you know that great mathematicians throughout history argued about the very existence of negative numbers? William Frend, a 16th-century European mathematician, refused to accept the existence of negative numbers. In his book *The Principles of Algebra* (1796), he wrote:

“To attempt to take [a number] away from a number less than itself is ridiculous.”

Even Augustus DeMorgan, author of the famed DeMorgan Theorems, thought that numbers less than zero were unimaginable. We all know now that negative numbers do exist. We learned about them in the third grade, and we use them every day. A golfer who scores a 67 on a par 72 course would describe her score as 5 under par, or –5. Likewise, in the northern climate of the United States, the winter temperatures can drop to 10° below 0, or –10 °Fahrenheit. If negative decimal numbers exist and you can convert a decimal number into its binary equivalent, then there must be a way to represent negative binary numbers. In this activity you will learn how to express numbers in their 8-bit - 2’s complement binary equivalent. You will use these equivalencies to perform simple addition and subtraction.

1. Express the following **decimal numbers** as their **8-bit - 2’s complement binary equivalent**.
   a. $114_{10} =$
   b. $-49_{10} =$
   c. $87_{10} =$
   d. $-108_{10} =$
   e. $-97_{10} =$
2. Express the following **8-bit, 2’s complement binary number** as their **decimal equivalent**.
   a. $11011001_2 =$
   b. $01110100_2 =$
   c. $10110001_2 =$
   d. $10111101_2 =$
   e. $00011011_2 =$
3. Perform each of the following additions in 2’s complement form. Check your answers by converting the 2’s complement binary numbers into their decimal equivalents and adding.

a.
$$\begin{array}{r} 0\ 0\ 0\ 1\ 0\ 1\ 1\ 0 \\ +\ 0\ 0\ 1\ 1\ 0\ 1\ 0\ 1 \\ \hline \end{array}$$

b.
$$\begin{array}{r} 0\ 1\ 1\ 1\ 0\ 1\ 0\ 0 \\ +\ 1\ 0\ 1\ 0\ 1\ 0\ 1\ 1 \\ \hline \end{array}$$

c.
$$\begin{array}{r} 1\ 0\ 0\ 1\ 1\ 1\ 1\ 0 \\ +\ 0\ 0\ 1\ 0\ 1\ 0\ 1\ 1 \\ \hline \end{array}$$

d.
$$\begin{array}{r} 1\ 1\ 0\ 1\ 1\ 1\ 0\ 1 \\ +\ 1\ 1\ 1\ 0\ 0\ 0\ 1\ 1 \\ \hline \end{array}$$

4. After converting each decimal number to its 8-bit - **2's complement** binary equivalent, perform the following arithmetic operations. Check your answers by converting the 2's complement binary numbers into their decimal equivalents.
   (Remember, subtraction is the same as addition of a negative number.)

   a.
$$\begin{array}{r} 23 \\ +\quad 14 \\ \hline \end{array}$$

   b.
$$\begin{array}{r} 36 \\ -\quad 28 \\ \hline \end{array}$$

   c.
$$\begin{array}{r} (-47) \\ +\quad 69 \\ \hline \end{array}$$

   d.
$$\begin{array}{r} 46 \\ +\ (-72) \\ \hline \end{array}$$

Answer the following in your engineering notebook:

5. What is the largest positive and smallest negative decimal number that can be expressed as an 8-bit, 2's complement binary number?

6. Without performing any binary-to-decimal conversions, which of the following binary numbers must be –49?
   00110001 11001111 01100110
   How were you able to determine this?